THERMAL RESONANCE IN A CLOSED CIRCULATION LOOP

A. A. Repin

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The results of an analytical investigation of transient heat transfer conditions in a closed circulation loop are given. They make it possible to determine the conditions of the resonance increase in the coolant temperature fluctuation amplitude as a function of the temperature variation frequency in the core and the hydrodynamic velocity of heat transfer along the loop.

The flow of one or several coolants in power plants constitutes recirculation flow in a closed circulation loop. Such flow features the specific characteristics of the transient conditions of heat transfer, including vibrational characteristics.

As is known, the heat transfer equations in themselves do not generate fluctuation processes. They develop in the presence of other phenomena – mechanical, electrical, optical, etc. – besides the heat transfer phenomena [1]. Mathematically, this means that the source of unsteadiness is to be found either in the supplementary equations or in the boundary conditions. Correspondingly, thermal fluctuations could be caused by changes in the thermal operating conditions in the core of the circulation loop, which are characterized by wide ranges of amplitudes and frequencies in, for instance, transport power plants.

Investigation of such transient operating conditions is of great practical importance in striving to ensure reliable functioning of elements in the heat transfer loop. The reason for this is that fluctuations of the coolant temperature are unavoidably accompanied by pressure vibrations, which affect the strength characteristics of pipes and heat exchangers. The pressure amplitude in the flow of liquid coolants is especially significant. For instance, the pressure amplitude is roughly proportional to the fourth power of the temperature amplitude (in degrees Celsius) in a water flow. Besides affecting the strength characteristics, transient pressure changes also influence the thermal efficiency of the loop. As was shown in [2], one of the causes of this is a more intensive evolution of the gaseous phase from the liquid flow. This reduces the effective cross section of the liquid phase of the flow, increases its velocity, and raises the hydraulic drag of the loop, and thus possibly reduces the circulation discharge of the coolant.

The development of the pressure vibrations caused by temperature fluctuations are different from the pressure changes due to self-oscillations in elements of the hydraulic systems and the vibrational loads affecting the supply mains, mentioned, for instance, in [2], or the thermohydraulic self-oscillations occurring during the circulation of two-phase coolants [3]. Thus, it must be the subject of special investigations.

As a first step, we shall consider below the problem of changes in the coolant temperature with temperature fluctuations in the active section of the circulation loop.

We assume that nonsteady heat supply is provided over the section $x \in (0, L_1)$ in a circulation loop whose length is L. The temperature of the external coolants in the core is equal to $T_a = T_0 + A_a \exp(i\omega t)$, while the heat transfer coefficient is k_a; heat is removed over the section $x \in (L_2, L_3)$ through a heat exchanger characterized by the heat transfer coefficient k_h and a constant temperature of the receiving coolant.

According to this model, the heat transfer in the loop is described by a system of adjoint equations, which are written in the well-known form (for instance, [4, 5]):

in the core ($0 < x < L_1$),

$$\frac{\partial T_1}{\partial t} + u\frac{\partial T_1}{\partial x} = \frac{k_a \Pi}{\rho c_p}(T_a - T_1),$$

All-Union Scientific-Research Institute of Transport Machine Construction, Leningrad.
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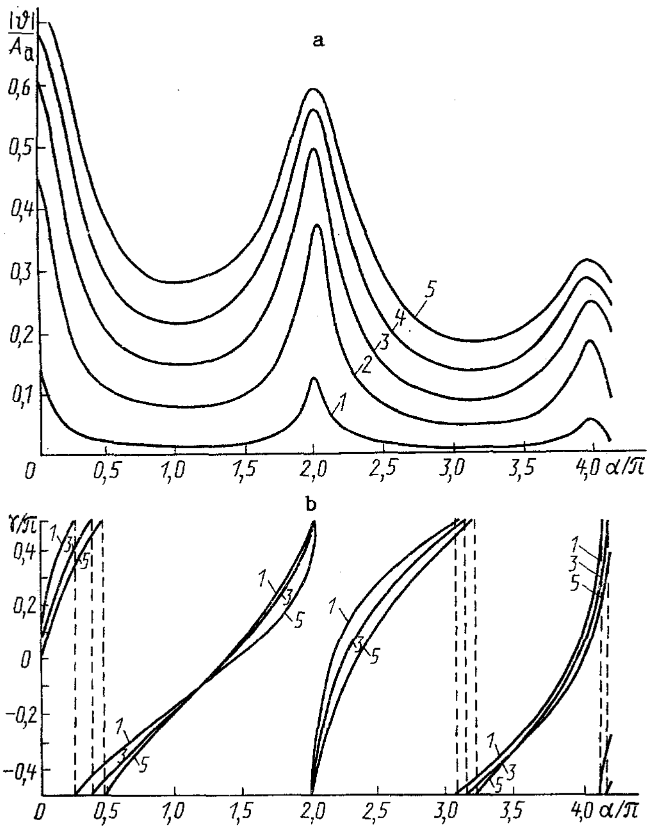

Fig. 1. Amplitude (a) and phase shift (b) of coolant temperature fluctuations as functions of the temperature fluctuation frequency in the core for $\ell_1 = \Delta\ell = 1/3$ and $b_h = 0.5$. 1) $b_a = 0.1$; 2) 0.5; 3) 1.0; 4) 1.5; 5) 2.0.

in the section of heat transfer from the core to the heat exchanger ($L_1 < x < L_2$)

$$\frac{\partial T_2}{\partial t} + u\frac{\partial T_2}{\partial x} = 0,$$

in the heat exchanger ($L_2 < x < L_3$):

$$\frac{\partial T_3}{\partial t} + u\frac{\partial T_3}{\partial x} = \frac{k_h \Pi}{\rho c_p}(T_h - T_3)$$

in the section of heat transfer from the heat exchanger to the core ($L_3 < x < L$):

$$\frac{\partial T_4}{\partial t} + u\frac{\partial T_4}{\partial x} = 0.$$

The adjunction of the above equations requires that the equations $T_1(L_1, t) = T_2(L_1, t)$, $T_2(L_2, t) = T_3(L_2, t)$, $T_3(L_3 t) = T_4(L_3, t)$, and $T_4(1, t) = T_1(0, t)$, be satistied. The latter ensure the continuity of the temperature distribution along the loop.

The system of equations can be replaced by a single equation with discontinuous coefficients, which is written as follows in the dimensionless form:

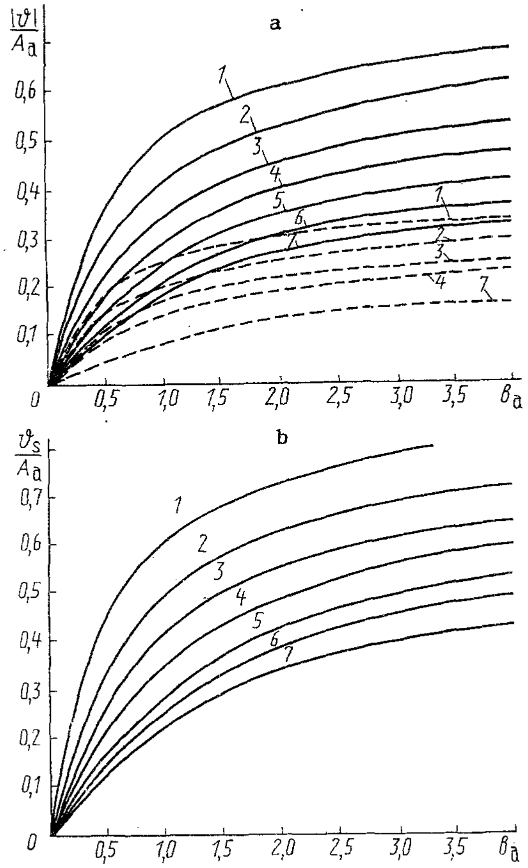

Fig. 2. Resonance (a) and quasi-steady (b) amplitudes of coolant temperature fluctuations as functions of the heat transfer coefficient for $\ell_1 = \Delta\ell = 1/3$. 1) $b_h = 0.5$; 2) 0.75; 3) 1.0; 4) 1.25; 5) 1.5; 6) 1.75; 7) 2.0.

$$a(\xi)\frac{\partial T}{\partial \tau} + \frac{\partial T}{\partial \xi} = b(\xi)\,[T_0(\xi) + A(\xi)\exp(i\alpha\tau) - T]. \quad (1)$$

It is assumed here that the mass discharge of the coolant $G = \rho fu$ is independent of the time and the coordinate. This would occur in the absence of coolant removal from the loop and a sufficiently steep pressure characteristic of the transfer pump, which would allow us to neglect the effect of loop pressure fluctuations on the circulation discharge.

The Cauchy problem is usually considered for Eq. (1). For slow dynamic processes, where the characteristic time of variation of the input parameters is longer than the relaxation time, the problem can be simplified considerably by taking into account the effect of the manner in which the input parameters vary on the output parameters, using functions with a lagging argument [6]. However, this approach cannot be used for a closed loop, where the initial Cauchy conditions must be replaced with the equation T(0, t) = T(L, t).

As far as we know, such a boundary-value problem has not been considered for first-order equations of the type (1). However, it has certain specific features; in particular, the function T(0, t) is not assigned, as in the Cauchy problem, but is uniquely determined from the condition T(0, t) = T(L, t). It is sometimes considered that such a problem is improper. However, this is not so, as far as one can judge from the solution given below; one of the characteristics of this solution is thermal resonance.

The solution of Eq. (1) is sought in the following form:

$$T = T_s(\xi) + \vartheta(\xi)\exp(i\alpha\tau) \text{ for } T(0, \tau) = T(1, \tau). \quad (2)$$

The steady-state component in (2) satisfies the equation

$$\frac{dT_s}{d\xi} = b(\xi)\,[T_0(\xi) - T_s(\xi)], \tag{3}$$

the solution of which is given by

$$T_s(\xi) = \exp(-\beta)\left[B + \int_0^{\xi} b(\xi)\,T_0(\xi)\exp\beta d\xi\right], \tag{4}$$

where $\beta(\xi) = \int_0^{\xi} b(\xi)\,d\xi$, while the constant B, determined from the condition $T_s(0) = T_s(1)$, is equal to

$$B = \int_0^{1} b(\xi)\,T_0(\xi)\exp\beta d\xi\,[\exp\beta(1) - 1]^{-1}. \tag{5}$$

The amplitude of the transient component is determined by solving the equation

$$\frac{d\vartheta}{d\xi} + \vartheta\,[i\alpha a(\xi) + b(\xi)] = b(\xi)\,A(\xi) \tag{6}$$

and is found to be

$$\vartheta = \exp[-(\beta + i\alpha a_0)]\left[C + \int_0^{\xi} b(\xi)\,A(\xi)\exp(\beta + i\alpha a_0)\,d\xi\right], \tag{7}$$

where $a_0 = \int_0^{\xi} a(\xi)\,d\xi$, and

$$C = \int_0^{1}\exp(\beta + i\alpha a_0)\,b(\xi)\,A(\xi)\,d\xi\,[\exp(\beta(1) + i\alpha a_0(1)) - 1]^{-1}. \tag{8}$$

Denoting $J_1(\xi) = \int_0^{\xi}\cos(\alpha a_0)\exp\beta d\xi$, and $J_2(\xi) = \int_0^{\xi}\sin(\alpha a_0)\exp\beta d\xi$ and separating the real and the imaginary parts in $\vartheta = \vartheta_r + i\vartheta_i$, and $C = C_r + iC_i$, we find

$$\vartheta_r = [C_r + J_1(\xi)]\cos(\alpha a_0)\exp(-\beta) + [C_i + J_2(\xi)]\sin(\alpha a_0)\exp(-\beta), \tag{9}$$

$$\vartheta_i = [C_i + J_2(\xi)]\cos(\alpha a_0)\exp(-\beta) - [C_r + J_1(\xi)]\sin(\alpha a_0)\exp(-\beta), \tag{10}$$

$$C_r = \frac{J_1(1)\,[\cos\alpha a_0(1)\exp\beta(1) - 1] + J_2(1)\sin\alpha a_0(1)\exp\beta(1)}{\exp 2\beta(1) - 2\exp\beta(1)\cos\alpha a_0(1) + 1}, \tag{11}$$

$$C_i = \frac{J_2(1)\,[\cos\alpha a_0(1)\exp\beta(1) - 1] - J_1(1)\sin\alpha a_0(1)\exp\beta(1)}{\exp 2\beta(1) - 2\exp\beta(1)\cos\alpha a_0(1) + 1}. \tag{12}$$

In accordance with (4), (5), and (9)-(12), the solution of Eq. (1) is written as follows:

$$T = T_s(\xi) + |\vartheta|\exp[i(\alpha\tau + \gamma - \alpha a_0)], \tag{13}$$

where the absolute value of the amplitude is equal to

$$\vartheta = \sqrt{\vartheta_r^2 + \vartheta_i^2} = \exp[-\beta(\xi)]\sqrt{[C_r + J_1(\xi)]^2 + [C_i + J_2(\xi)]^2}; \tag{14}$$

the phase shift due to convective heat transfer is represented by αa_0; the phase shift connected with the superposition of the waves propagating along the loop is determined by the expression

$$\gamma = \mathrm{arctg}\,\{[C_i + J_2(\xi)]/[C_r + J_1(\xi)]\}. \tag{15}$$

Using the general equations (9)-(15), we write the expression for $|\vartheta|$ and γ specifically for a contour with a constant transverse cross section ($a = 1$), bearing in mind that the functions $b(\xi)$, $A(\xi)$, $\beta(\xi)$, and $a_0(\xi)$ are determined by the following relationships: $0 < \xi < \ell_1$, $b = b_a$, $A = A_a$, $\beta = b_a\xi$, $a_0 = \xi$; $\ell_1 < \xi < \ell_2$, $b = 0$, $A = 0$, $\beta = b_a\ell_1$, $\alpha_0 = \xi$; $\ell_2 < \xi < \ell_3$, $b = b_h$, $A = 0$, $\beta = b_a\ell_1 + b_h(\xi-\ell_2)$, $a_0 = \xi$; $\ell_3 < \xi < 1$, $b = 0$, $A = 0$, $\beta = b_a \times \ell_1 + b_h\Delta\ell$, $\alpha_0 = \xi$.

By substituting these functions in the integrals $J_1(\xi)$ and $J_2(\xi)$ and further in (14) and (15) with an allowance for (9)-(12), we obtain the following expressions for the relative amplitude and the phase shift at $\xi = 0$:

$$\frac{|\vartheta|}{A_a} = \frac{b_a}{\sqrt{b_a^2 + \alpha^2}} \sqrt{\frac{\exp(2b_a l_1) - 2\exp(b_a l_1)\cos(\alpha l_1) + 1}{\exp 2\beta(1) - 2\exp\beta(1)\cos\alpha + 1}}, \tag{16}$$

$$\mathrm{tg}\,\gamma = \frac{J_1(1)\exp\beta(1)\sin\alpha - J_2(1)[\exp\beta(1)\cos\alpha - 1]}{J_1(1)[\exp\beta(1)\cos\alpha - 1] + J_2(1)\exp\beta(1)\sin\alpha}. \tag{17}$$

It is evident from (16) and (17) that the dependences of $|\vartheta|$ and γ on α are not monotonic functions. For $\alpha = 2\pi n$, the denominator in the radicant of (16) assumes a minimum value, and the amplitude correspondingly assumes the maximum value. The dependence of $|\vartheta|$ on α is given in Fig. 1a, while γ as a function of α is shown in Fig. 1b. The sharp rise of $|\vartheta|$ at $\alpha = 2\pi$ is noteworthy; there is also traced the second maximum at $\alpha = 4\pi$, but it is roughly half as large as the first one.

The above values of α can be termed the resonance points. Passing to the dimensional linear frequency $\nu = \omega/2\pi$ and recalling the definition $\alpha = \omega L/U$, we conclude that the condition of resonance consists in the multiplicity of the frequency ν of fluctuations in the heat flux supplied and of the frequency U/L of coolant circulation in the loop. This phenomenon is similar to the resonance in forced vibrations of a mechanical system when its natural frequency coincides with the frequency of the perturbation force.

The phase shift γ depends basically on α; at resonance values, α changes its sign abruptly and is not described by simple relaxation relationships with a lagging argument, as in [6].

Thus, we have established the nontrivial, resonance character of coolant temperature variation in a closed circulation loop with periodic changes of the thermal flux in the core.

This phenomenon can be utilized for many practical purposes, in particular, measurements in heat engineering involving transient phenomena. Actually, direct measurement of the transient temperature, the thermal flux, or its fluctuation frequency in the heat generation zone often presents technical difficulties. However, there is now a possibility of determining these quantities in a simpler way on the basis of the derived relationships between $|\vartheta|$ and γ on the one hand, and α on the other. For this, it is sufficient to measure the temperatures and the circulation discharge of the coolant at accessible points of the loop. For this, we must know the reduced heat transfer coefficients b_a and b_h, since they affect materially the temperature amplitude. The character of this influence is evident from the curves in Fig. 2a, which indicate that the amplitude increases with b_a and decreases with an increase in b_h. Hence it follows, in particular, the important conclusion that the heat exchanger in the loop damps the thermal flux oscillations arising in the heat generation zone, the more so, the higher the heat exchanger efficiency. To a certain extent, this is similar to the effect of friction forces in forced vibrations of mechanical systems.

The values of b_a and b_h can be determined either by calculation or experimentally for actual power plants on the basis of temperature measurements under steady-state conditions ($\partial/\partial\tau = 0$, $\alpha = 0$).

Actually, in correspondence with (4) and (5), and assuming that $T_0(\xi) = T_a$ for $0 < \xi < \ell_1$ and $T_0(\xi) = T_h$ for $\ell_2 < \xi < \ell_3$, we have

$$T_s(0) = T_h + (T_a - T_h)\frac{\exp(b_a l_1) - 1}{\exp(b_a l_1 + b_h \Delta l) - 1}, \tag{18}$$

$$T_s(l_1) = T_a - [T_a - T_s(0)]\exp(-b_a l_1). \tag{19}$$

Transforming the expressions (18) and (19), we obtain

$$b_a l_1 = \ln\frac{T_a - T_s(0)}{T_a - T_s(l_1)}, \tag{20}$$

$$b_h \Delta l = \ln\left[\frac{T_a - T_s(l_1)}{T_a - T_s(0)} + \frac{T_s(l_1) - T_s(0)}{T_s(0) - T_h}\,\frac{T_a - T_h}{T_a - T_s(0)}\right]. \tag{21}$$

Thus, for determining b_a and b_h, it is sufficient to measure the temperatures T_a, T_h, $T(0)$, and $T(\ell_1)$ under steady-state conditions. Subsequently, the found values of b_a and b_h can be used for analyzing and measuring the transient processes on the basis of relationships (16) and (17).

Let us now consider the relationship between the actual amplitudes of temperature fluctuations and the amplitudes determined by using the quasi-steady method ($\partial/\partial\tau = 0$). Under quasi-steady conditions, the relative amplitude is determined by the expression

$$\frac{\vartheta_s}{A_a} = \frac{T_s(0) - T_h}{T_a - T_h} = \frac{\exp(b_a l_1) - 1}{\exp(b_a l_1 + b_h \Delta l) - 1} \tag{22}$$

(see Fig. 2b).

The resonance amplitudes and the phase shift γ in the case of full resonance, which occurs when $\cos\alpha = 1$ and $\cos\alpha\ell_1 = -1$, i.e., $\ell_1 = 1/2$ for $\alpha = 2\pi$, are described by the expressions

$$\frac{|\vartheta|}{A_a} = \frac{b_a}{\sqrt{b_a^2 + 4\pi^2}}\,\frac{\exp(b_a/2) + 1}{\exp(b_a/2 + b_h \Delta l) - 1}, \tag{23}$$

$$\gamma = -\operatorname{arctg}(2\pi/b_a). \tag{24}$$

Comparing (22) and (23), we obtain the expression

$$\frac{|\vartheta|}{\vartheta_s} = \frac{b_a}{\sqrt{b_a^2 + 4\pi^2}}\operatorname{cth}(b_a/2), \tag{25}$$

from which follows that $|\vartheta| < \vartheta_s$ always. Thus, for actual transient processes, estimates based on quasi-steady conditions cannot be used.

All these specific features, found for $\xi = 0$, occur also at any point of the loop ($\xi \neq 0$). Under full resonance conditions, the numerical values of $|\vartheta|/A_a$ are independent of ξ and are determined by expression (23). The difference consists only in the value of the phase shift in (13) because of the different values of $a_0(\xi)$.

These cases of resonance considered above also occur when the fluctuation mode of the thermal flux is different from that used in Eq. (1). In particular, temperature fluctuations with an infinite amplitude arise if the thermal flux in the core ($0 < \xi < \ell_1$) is independent of the coolant temperature. Actually, the heat transfer equation is in this case written thus:

$$a(\xi)\frac{\partial T}{\partial\tau} + \frac{\partial T}{\partial\xi} = b(\xi)\,[T_0(\xi) + A(\xi)\exp(i\alpha\tau)]. \tag{1a}$$

The solution for the transient component in (1a) can be obtained from (7) for $\beta = 0$; as a result, in correspondence with (16), the amplitude is infinite at the resonance points.

The solutions obtained can also be used for analyzing the relaxation processes. The importance of this analysis is related to the fact that, in practical work, the state of the core is usually estimated with respect to the thermal state of the coolant in the loop ($0 < \xi < \ell_1$) by utilizing, for instance, expression (18) and (19) for steady-state conditions. However, this cannot be done for transient processes because of the different rates of temperature variation in the coolant and the core. Acceptable coolant temperatures still do not indicate a permissible thermal state of the core.

Assuming that $\alpha = -i\alpha_0$, in (1), where α_0 is the real value, we have an exponential temperature rise in the core: $T_a = T_0 + A_a \exp(\alpha_0\tau)$. In this case, the coolant temperature also increases exponentially; in correspondence with (7) and (8), we obtain the following expression for the preexponential factor at $\xi = 0$:

$$\frac{\vartheta(0)}{A_a} = \frac{b_a}{b_a + \alpha_0} \frac{\exp[(b_a + \alpha_0) l_1] - 1}{\exp[b_a l_1 + b_h \Delta l + \alpha_0] - 1}. \quad (26)$$

It follows from (26) that, for large values of α_0 corresponding to a high rate of increase in T_a, the values of $\vartheta(0)$ are much smaller than A_a. This indicates that, at the initial instants of time, the coolant reacts slightly to changes in T_a. Therefore, in transient heating, the variation of $T = \vartheta \exp(\alpha_0\tau)$ must be monitored carefully with stepped-up accuracy on the basis of expression (26) in order to predict critical conditions in the core.

Subsequently, the expressions derived for transient temperature variation can be used in the hydrodynamic equations for the circulation loop in order to determine the coolant pressure as a function of the temperature and, thus, the state of mechanical strength of the mains and the heat exchangers.

NOTATION

T, temperature; A_a and ϑ, temperature variation amplitudes; x and $\xi = x/L$, dimensional and dimensionless coordinates along a circulation loop whose length is L, respectively; t and $\tau = tU/L$, dimensional and dimensionless time, respectively; U, characteristic flow velocity in the circulation loop; ω, temperature variation frequency; $\alpha = \omega L/U$; $a(\xi) = f(\xi)/L$, dimensionless cross-sectional area of the loop; F, characteristic area; k, heat transfer coefficient; $b = k\Pi L/C_p \rho f u$, reduced heat transfer coefficient; u, local velocity of the coolant; Π, segment of the channel perimeter through which heat transfer occurs; ρ and c_p; density and specific heat of the coolant, respectively; $G = \rho f u$, circulation discharge of $a_0(\xi) = \int_0^\xi a(\xi) d\xi$, and $\beta(\xi) = \int_0^\xi b(\xi) d\xi$, reduced coefficients; $J_1(\xi)$ and $J_2(\xi)$, auxiliary functions; γ, phase shift; C_r and C_i, integration constants; ℓ_1, length of the loop segment comprising the heat-supplying core; $\Delta\ell = \ell_3 - \ell_2$, length of the loop segment where heat is removed. Subscripts: a and h pertain to the temperature and the heat transfer coefficient in the core and in the segment of heat removal, respectively; s: steady-state conditions; r and i: real and imaginary parts, respectively.

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